Calculus Assignment Tangents and normal s and there equation,MS

**1a.** *[3 marks]*

Markscheme

 ***A1A1***

correct substitution ***A1***

*eg*

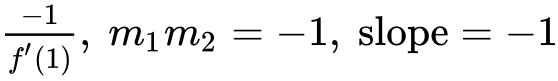
 ***AG N0***

***[3 marks]***

**1b.** *[3 marks]*

Markscheme

correct approach to find the gradient of the normal ***(A1)***

*eg*

attempt to substitute correct normal gradient and coordinates into equation of a line ***(M1)***

*eg*

 ***A1 N2***

***[3 marks]***

**1c.** *[4 marks]*

Markscheme

equating expressions ***(M1)***

*eg*

correct working (must involve combining terms) ***(A1)***

*eg*

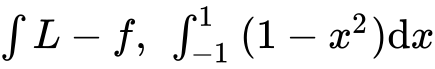
 ***A2 N3***

***[4 marks]***

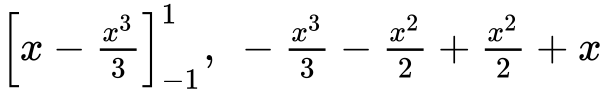
**1d.** *[6 marks]*

Markscheme

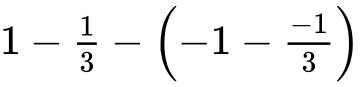
valid approach ***(M1)***

*eg*, splitting area into triangles and integrals

correct integration ***(A1)(A1)***

*eg*

substituting **their** limits into **their** integrated function and subtracting (in any order) ***(M1)***

*eg*

**Note:** Award ***M0*** for substituting into original or differentiated function.

area  ***A2 N3***

***[6 marks]***

**2a.** *[1 mark]*

Markscheme

 ***A1*** ***N1***

***[1 mark]***

**2b.** *[3 marks]*

Markscheme

gradient of normal  ***(A1)***

attempt to substitute their normal gradient and coordinates of P (in any order) ***(M1)***

*eg*

 ***A1*** ***N3***

***[3 marks]***

**2c.** *[2 marks]*

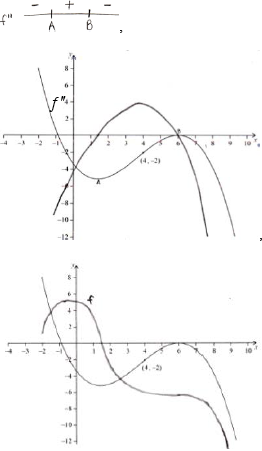
Markscheme

correct answer **and** valid reasoning ***A2*** ***N2***

answer: *eg* graph of  is concave up, concavity is positive (between )

reason: *eg* slope of  is positive,  is increasing, ,

sign chart (must clearly be for  and show A and B)



**Note:** The reason given must refer to a specific function/graph. Referring to “the graph” or “it” is not sufficient.

***[2 marks]***

**3a.** *[3 marks]*

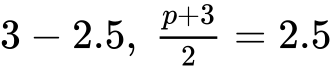
Markscheme

**METHOD 1 (using *x*-intercept)**

determining that 3 is an -intercept ***(M1)***

*eg*, 

valid approach ***(M1)***

*eg*

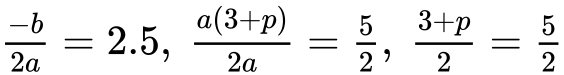
 ***A1*** ***N2***

**METHOD 2 (expanding *f* (*x*))**

correct expansion (accept absence of ) ***(A1)***

*eg*

valid approach involving equation of axis of symmetry ***(M1)***

*eg*

 ***A1*** ***N2***

**METHOD 3 (using derivative)**

correct derivative (accept absence of ) ***(A1)***

*eg*

valid approach **(*M1)***

*eg*

 ***A1*** ***N2***

***[3 marks]***

**3b.** *[3 marks]*

Markscheme

attempt to substitute  ***(M1)***

*eg*

correct working ***(A1)***

*eg*

 ***A1*** ***N2***

***[3 marks]***

**3c.** *[8 marks]*

Markscheme

**METHOD 1 (using discriminant)**

recognizing tangent intersects curve once ***(M1)***

recognizing one solution when discriminant = 0 ***M1***

attempt to set up equation ***(M1)***

*eg*

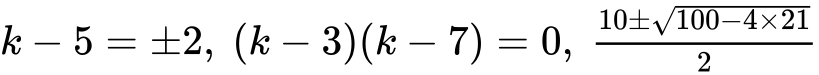
rearranging their equation to equal zero ***(M1)***

*eg*

correct discriminant (if seen explicitly, not just in quadratic formula) ***A1***

*eg*

correct working ***(A1)***

*eg*

 ***A1A1*** ***N0***

**METHOD 2 (using derivatives)**

attempt to set up equation ***(M1)***

*eg*

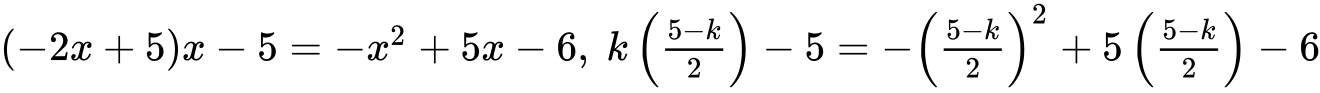
recognizing derivative/slope are equal ***(M1)***

*eg*

correct derivative of  ***(A1)***

*eg*

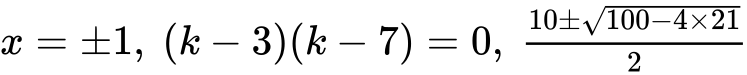
attempt to set up equation in terms of either  or  ***M1***

*eg*

rearranging their equation to equal zero ***(M1)***

*eg*

correct working ***(A1)***

*eg*

 ***A1A1*** ***N0***

***[8 marks]***

**4a.** *[1 mark]*

Markscheme

 ***A1*** ***N1***

***[1 mark]***

**4b.** *[2 marks]*

Markscheme

attempt to substitute  into their derivative ***(M1)***

gradient of  is  ***A1*** ***N2***

***[2 marks]***

**4c.** *[5 marks]*

Markscheme

**METHOD 1**

attempt to substitute coordinates of A and their gradient into equation of a line ***(M1)***

*eg*

correct equation of  in any form ***(A1)***

*eg*

valid approach ***(M1)***

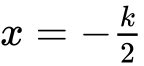
*eg*

correct substitution into  equation ***A1***

*eg*

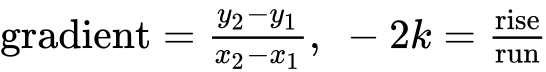
correct working ***A1***

*eg*

 ***AG*** ***N0***

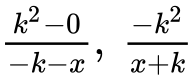
**METHOD 2**

valid approach ***(M1)***

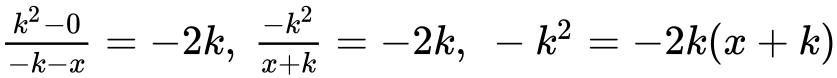
*eg*

recognizing  at B ***(A1)***

attempt to substitute coordinates of A and B into slope formula ***(M1)***

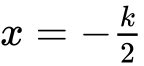
*eg*

correct equation ***A1***

*eg*

correct working ***A1***

*eg*

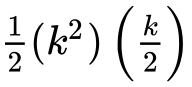
 ***AG*** ***N0***

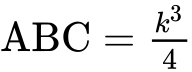
***[5 marks]***

**4d.** *[2 marks]*

Markscheme

valid approach to find area of triangle ***(M1)***

*eg*

area of  ***A1*** ***N2***

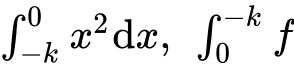
***[2 marks]***

**4e.** *[7 marks]*

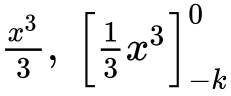
Markscheme

**METHOD 1 (****)**

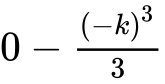
valid approach to find area from  to 0 ***(M1)***

*eg*

correct integration (seen anywhere, even if ***M0*** awarded) ***A1***

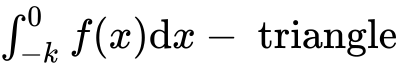
*eg*

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

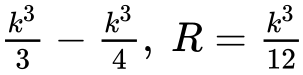
*eg*, area from  to 0 is 

**Note:** Award ***M0*** for substituting into original or differentiated function.

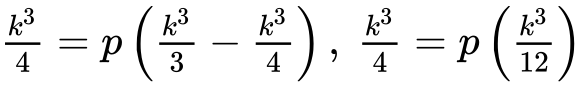
attempt to find area of  ***(M1)***

*eg*

correct working for  ***(A1)***

*eg*

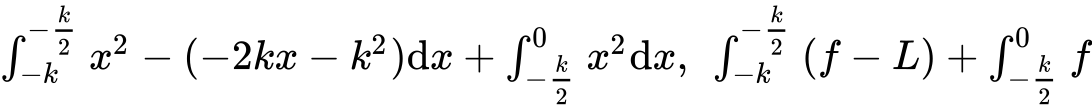
correct substitution into  ***(A1)***

*eg*

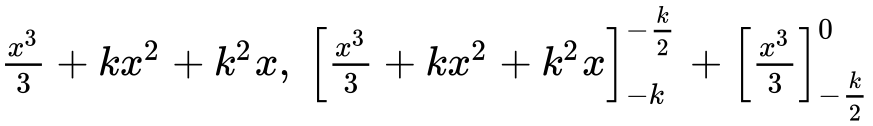
 ***A1*** ***N2***

**METHOD 2 (****)**

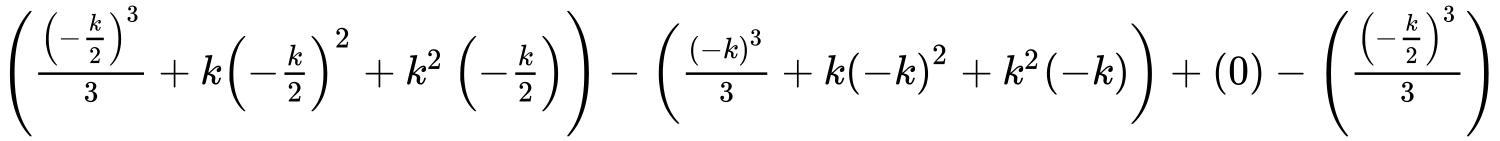
valid approach to find area of  ***(M1)***

*eg*

correct integration (seen anywhere, even if ***M0*** awarded) ***A2***

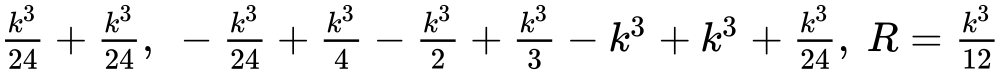
*eg*

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

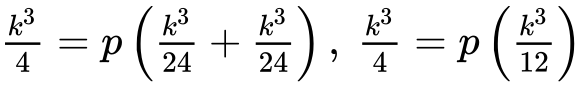
*eg*

**Note:** Award ***M0*** for substituting into original or differentiated function.

correct working for  ***(A1)***

*eg*

correct substitution into  ***(A1)***

*eg*

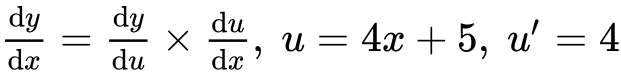
 ***A1*** ***N2***

***[7 marks]***

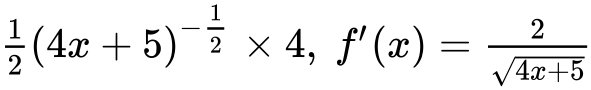
**5a.** *[4 marks]*

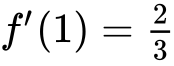
Markscheme

choosing chain rule ***(M1)***

*eg*

correct derivative of  ***A2***

*eg*

 ***A1 N2***

***[4 marks]***

**5b.** *[2 marks]*

Markscheme

recognize that  is the gradient of the tangent ***(M1)***

*eg*

 ***A1 N2***

***[2 marks]***

**5c.** *[2 marks]*

Markscheme

recognize that R is on the tangent ***(M1)***

*eg*, sketch

 ***A1 N2***

***[2 marks]***

**5d.** *[7 marks]*

Markscheme

 (seen anywhere) ***A1***

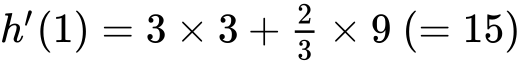
 (seen anywhere) ***A1***

choosing product rule to find  ***(M1)***

*eg*

correct substitution to find  ***(A1)***

*eg*

 ***A1***

**EITHER**

attempt to substitute coordinates (in any order) into the equation of a straight line ***(M1)***

*eg*

 ***A1 N2***

**OR**

attempt to substitute coordinates (in any order) to find the -intercept ***(M1)***

*eg*

 ***A1 N2***

***[7 marks]***

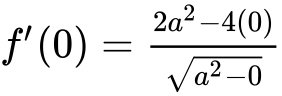
**6a.** *[6 marks]*

Markscheme

(i) recognizing the need to find the gradient when  (seen anywhere) ***R1***

*eg*

correct substitution ***(A1)***



 ***(A1)***

correct equation with gradient 2 (do not accept equations of the form ) ***A1 N3***

*eg*

(ii) **METHOD 1**

attempt to substitute  into **their** equation of  ***(M1)***

*eg*

 ***A1 N2***

**METHOD 2**

equating gradients ***(M1)***

*eg*

 ***A1 N2***

***[6 marks]***

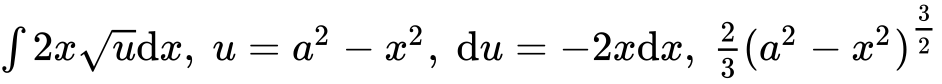
**6b.** *[6 marks]*

Markscheme

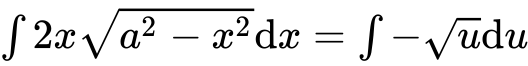
**METHOD 1**

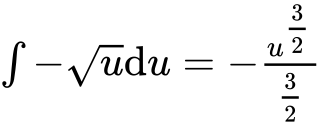
recognizing that area  (seen anywhere) ***R1***

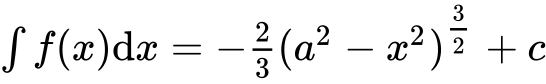
valid approach using substitution or inspection ***(M1)***

*eg*

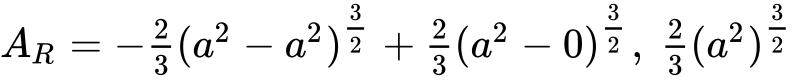
correct working ***(A1)***

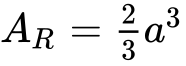
*eg*

 ***(A1)***

 ***(A1)***

substituting limits and subtracting ***A1***

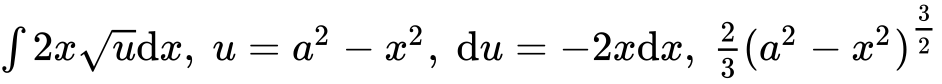
*eg*

 ***AG N0***

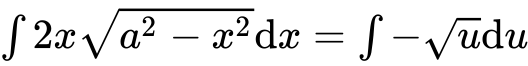
**METHOD 2**

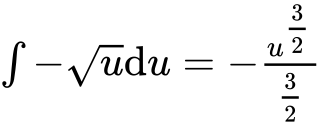
recognizing that area  (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

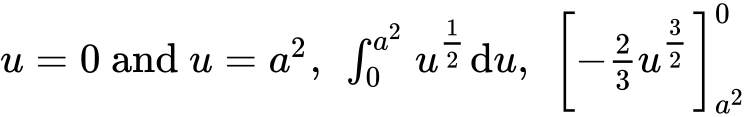
*eg*

correct working ***(A1)***

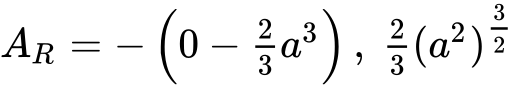
*eg*

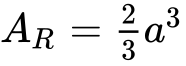
 ***(A1)***

new limits for *u* (even if integration is incorrect) ***(A1)***

*eg*

substituting limits and subtracting ***A1***

*eg*

 ***AG N0***

***[6 marks]***

**6c.** *[4 marks]*

Markscheme

**METHOD 1**

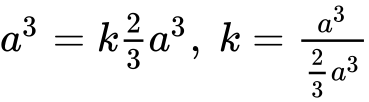
valid approach to find area of triangle ***(M1)***

*eg*

correct substitution into formula for  (seen anywhere) ***(A1)***

*eg*

valid attempt to find  (must be in terms of ) ***(M1)***

*eg*

 ***A1 N2***

**METHOD 2**

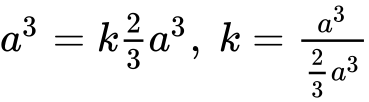
valid approach to find area of triangle ***(M1)***

*eg*

correct working ***(A1)***

*eg*

valid attempt to find  (must be in terms of ) ***(M1)***

*eg*

 ***A1 N2***

***[4 marks]***

**7a.** *[2 marks]*

Markscheme

**METHOD 1**

 ***(A1)***

valid reasoning including reference to the graph of  ***R1***

*eg* changes sign from negative to positive at , labelled sign chart for 

so  has a local minimum at  ***AG N0***

**Note:** It must be clear that any description is referring to the graph of , simply giving the conditions for a minimum without relating them to  does not gain the ***R1***.

**METHOD 2**

 ***A1***

valid reasoning referring to second derivative ***R1***

*eg*

so  has a local minimum at  ***AG N0***

***[2 marks]***

**7b.** *[2 marks]*

Markscheme

attempt to find relevant interval ***(M1)***

*eg* is decreasing, gradient of  is negative, 

(accept “between 2 and 4”) ***A1 N2***

**Notes:** If no other working shown, award ***M1A0*** for incorrect inequalities such as    4, or “from 2 to 4”

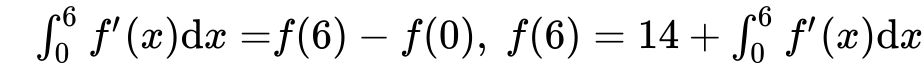
***[2 marks]***

**7c.** *[5 marks]*

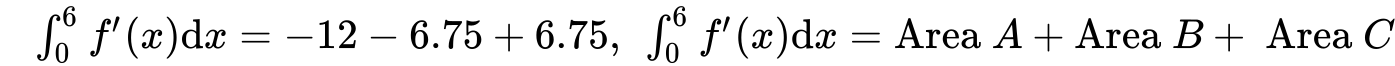
Markscheme

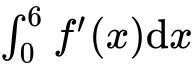
**METHOD 1 (one integral)**

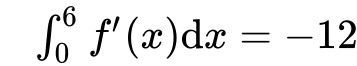
correct application of Fundamental Theorem of Calculus ***(A1)***

*eg*

attempt to link definite integral with areas ***(M1)***

*eg*

correct value for  ***(A1)***

*eg*

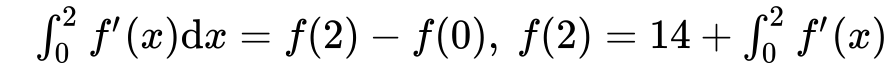
correct working ***A1***

*eg*

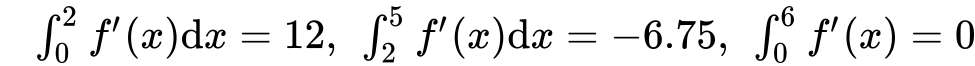
 ***A1 N3***

**METHOD 2 (more than one integral)**

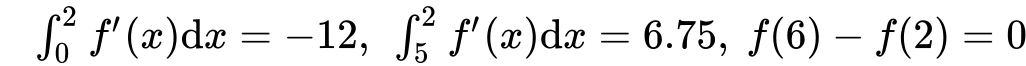
correct application of Fundamental Theorem of Calculus ***(A1)***

*eg*

attempt to link definite integrals with areas ***(M1)***

*eg*

correct values for integrals ***(A1)***

*eg*

one correct intermediate value ***A1***

*eg*

 ***A1 N3***

***[5 marks]***

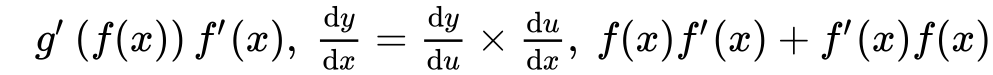
**7d.** *[6 marks]*

Markscheme

correct calculation of  (seen anywhere) ***A1***

*eg*

choosing chain rule or product rule ***(M1)***

*eg*

correct derivative ***(A1)***

*eg*

correct calculation of  (seen anywhere) ***A1***

*eg*

attempt to substitute **their** values of  and  (in any order) into equation of a line ***(M1)***

*eg*

correct equation in any form ***A1 N2***

*eg*

***[6 marks]***

***[Total 15 marks]***

**8a.** *[2 marks]*

Markscheme

 ***A1A1 N2***

***[2 marks]***

**8b.** *[3 marks]*

Markscheme

substituting  into  ***(M1)***

*eg*

recognizing (seen anywhere) ***M1***

correct equation ***A1***

*eg*

 ***AG N0***

***[3 marks]***

**8c.** *[2 marks]*

Markscheme

correct substitution into  ***(A1)***

*eg*

 ***A1 N2***

***[2 marks]***

**8d.** *[4 marks]*

Markscheme

recognizing gradient value (may be seen in equation) ***M1***

*eg*

attempt to substitute  into equation of a straight line ***M1***

*eg*

correct working ***(A1)***

*eg*

 ***A1 N2***

***[4 marks]***

**8e.** *[3 marks]*

Markscheme

**METHOD 1** ( derivative)

recognizing (seen anywhere) ***R1***

substituting  into  ***(M1)***

*eg*

 ***A1***

therefore the graph of  has a local maximum when  ***AG N0***

**METHOD 2** ( derivative)

recognizing change of sign of (seen anywhere) ***R1***

*eg**sign chart*

correct value of  for  ***A1***

*eg*

correct value of  for  value to the left of  ***A1***

*eg*

therefore the graph of  has a local maximum when  ***AG N0***

***[3 marks]***

***Total [14 marks]***

**9.** *[6 marks]*

Markscheme

recognising need to differentiate (seen anywhere) ***R1***

*eg* 

attempt to find the gradient when  ***(M1)***

*eg* 

 ***(A1)***

attempt to substitute coordinates (in any order) into equation of a straight line ***(M1)***

*eg* 

correct working ***(A1)***

*eg* 

 ***A1 N3***

***[6 marks]***

**10a.** *[3 marks]*

Markscheme

 ,  ,  ***A1A1A1 N3***

***[3 marks]***

**10b.** *[2 marks]*

Markscheme

 ***(A1)***

valid reasoning ***R1***

*eg*   changes sign at  , change in concavity of  at 

so P is a point of inflexion ***AG N0***

***[2 marks]***

**10c.** *[2 marks]*

Markscheme

writing  as a product of  and  ***A1***

*eg*   , 

 ***A1 N1***

***[2 marks]***

**10d.** *[7 marks]*

Markscheme

recognizing need to find derivative of  ***(R1)***

*eg*   , 

attempt to use the product rule (do **not** accept  ) ***(M1)***

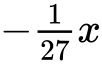
*eg*   , 

correct substitution ***(A1)***

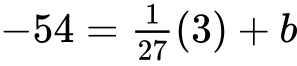
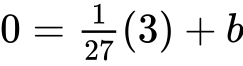
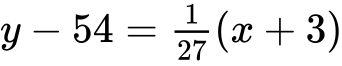
*eg*  

 ***A1***

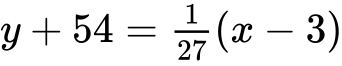
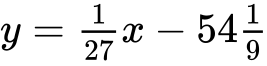
attempt to find the gradient of the normal ***(M1)***

*eg*   , 

attempt to substitute **their** coordinates and **their** normal gradient into the equation of a line ***(M1)***

*eg*   ,  ,  , 

correct equation in any form ***A1 N4***

*eg*   , 

***[7 marks]***

**11a.** *[3 marks]*

Markscheme

 ***A1A1A1 N3***

**Note**: Award ***A1*** for each term.

***[3 marks]***

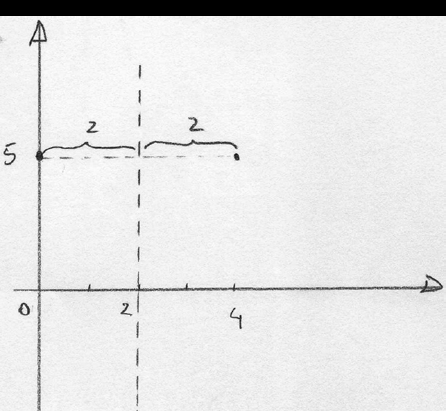
**11b.** *[3 marks]*

Markscheme

recognizing  gives the point (, ) ***(R1)***

recognize symmetry  ***(M1)***

*eg* vertex, sketch



 ***A1 N3***

***[3 marks]***

**11c.** *[4 marks]*

Markscheme

(i)   ***A1 N1***

(ii) substituting into  (not the vertex) ***(M1)***

*eg*   , 

working towards solution ***(A1)***

*eg*   , 

 ***A1 N2***

***[4 marks]***

**11d.** *[6 marks]*

Markscheme



correct derivative of  ***A1A1***

*eg*   , 

evidence of equating both derivatives ***(M1)***

*eg*  

correct equation ***(A1)***

*eg*  

working towards a solution ***(A1)***

*eg*   , combining like terms

 ***A1 N0***

**Note**: Do not award final ***A1*** if additional values are given.

***[6 marks]***

**12a.** *[1 mark]*

Markscheme

 ***A1 N1***

***[1 mark]***

**12b.** *[4 marks]*

Markscheme

(i) evidence of valid approach ***(M1)***

e.g.  , 

correct manipulation ***A1***

e.g.  , 

 ***AG N0***

(ii) evidence of finding   ***(M1)***

e.g. 

 ***A1 N2***

***[4 marks]***

**12c.** *[1 mark]*

Markscheme

 ***A1 N1***

***[1 mark]***

**13a.** *[4 marks]*

Markscheme

finding  ***A1***

attempt to find  ***(M1)***

correct value  ***A1***

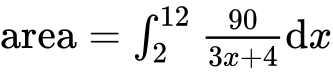
correct equation in any form ***A1 N2***

e.g.  , 

***[4 marks]***

**13b.** *[6 marks]*

Markscheme



correct integral ***A1A1***

e.g. 

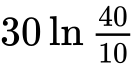
substituting limits and subtracting ***(M1)***

e.g.  , 

correct working ***(A1)***

e.g. 

correct application of  ***(A1)***

e.g. 

 ***A1 N4***

***[6 marks]***

**13c.** *[3 marks]*

Markscheme

valid approach  ***(M1)***

e.g. sketch, area *h* = area *g* , 120 + **their** answer from (b)

 ***A2 N3***

***[3 marks]***

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